Experimental Study on the Potential of Vehicle's Attitude Response to Railway Track Irregularity in Precise Train Localization

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Abstract-Railway track is never perfect, as rail distortions, namely, geometric irregularities, exist at all locations along the track. However, these distortions can be regarded as valuable indicators for train localization, since track irregularities present location-dependent characteristics, the measurements of which using onboard sensors are repeatable for the same track. In this research, we study the possibility of determining a train's position by matching the track irregularity measurements to a predefined map. A train-borne experiment on a real track is used to preliminarily demonstrate the feasibility, evaluate the performance and determine the key parameters for practical implementation. The results show that a submeter longitudinal localization accuracy can be achieved even when using a low-cost cabin-mounted microelectromechanical system (MEMS) inertial measurement unit (IMU), which measures the train's responses to track irregularities. The proposed method can enhance the positioning accuracy and improve the robustness of multisensory train localization systems.

Index Terms—Train localization, track irregularity matching, response to track irregularity, multisensory train positioning system.

I. INTRODUCTION

RAILWAY operations increasingly rely on real-time train position information to manage railway traffic. A train localization system estimates each train's location, direction and speed in the railway network. Numerous positioning sensors are available to determine the train location, which are usually classified into onboard sensors and infrastructure equipment by most authors [1]. Typical onboard sensors include tachometer [2], [3], odometer [4]–[7], satellite-based positioning system [8], [9], an inertial navigation system (INS) [10], [11], Doppler radar and vision-based positioning system, and typical infrastructure equipment include balise, track circuit and axle counter [1], [12], [13] etc. Each kind of sensor has its own advantages and shortcoming, we cannot

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use only a certain kind of sensor to achieve robust and reliable positioning solutions. For example, the global navigation satellite system (GNSS) presents limitations in signal-denied areas, in which case no GNSS solution is available. The positioning accuracy of an INS tends to drift with time when works standalone. Odometer or tachometer may be the most common sensors used in train positioning application, but they also suffer from a scale factor error. Therefore, these sensors are commonly used in combination, i.e., as a multisensory system, to achieve high levels of performance in terms of accuracy, integrity, reliability, continuity and availability. Of particular relevance to this work is the feature-matching-based train localization methods. Train positioning with onboard sensors and a track map is considered a base technology for future railway applications, such as train control without additional track-side infrastructures, train collision avoidance systems, and autonomous train driving [14].

The main contribution of the present paper is to propose a new kind of precise positioning signal, i.e., track irregularity, as part of an integrated system as will be described in section IV, and study its feasibility in matching-enabled train positioning through an experimental approach. Thus, in the following, our literature review concentrates on the featurematching-based localization researches.

The feature-matching-based localization approach determines each train's position by matching feature measurements to a predefined digital map that contains location-dependent features of interest [15]-[17]. Gerlach and zu Hörste [16] discussed the provision of a digital map for map matching. Saab [18], [19] proposed a map-matching algorithm for a train positioning system, which correlates the angular rates extracted from the design database to their corresponding measurements sensed by a yaw gyro and tachometer located on board the vehicle. This approach identifies position relative to a digital map within a few meters when significant curves are present. A limitation is that the beginning and end of the curves are often not very well defined and may move slightly with maintenance activities [20]. Many other feature-matching-based approaches exist, but similar to this example, the features of interest for matching are locationdependent geometric track features, including switches, mile posts, and other features of interest that can be found in previous studies [21], [22]. The features used for matching in these approaches have certain characteristics: they are discretely distributed along the track and generally have a

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Fig. 1. Concept of train localization by matching a track irregularity profile, taking the cross level as an example, to an onboard digital map.

distance interval of several kilometers. For these approaches, the track may become featureless along a long straight section when no significant curves exist, which limits the matching accuracy and availability.

Despite the progress being made in map matching and train localization, the accuracy that has been achieved is unlikely to be better than 1 m [20]. On the other hand, the accuracy requirement for future train localization systems is increasing. The purpose of the present research is to study a train localization method based on track irregularity feature matching. Similar to the work in [14], where a magnetic measurement profile from onboard magnetic sensors is matched to a digital magnetic map, the present work demonstrates the potential of a track irregularity map for real-time train localization with the goal of achieving submeter position accuracy in the longitudinal direction. Heirich et al. [14] noted that, in a more general way, any measurable signal that contains locationdependent information is suitable for navigation, provided that there is a methodology to extract this information [23]. Similar applications include terrain-based civil land vehicle localization using attitude measurements, which was performed by Dean et al. [24], [25]. That research inspired us to achieve precise train localization by matching a track irregularity profile to a digital map, as illustrated in Fig. 1. An important difference between the present research and previous studies is that continuous features, *i.e.*, an irregularity profile, are used instead of discretely placed features for matching, which offers the potential and possibility for precise localization.

Track irregularity, i.e., track deformation, refers to rails or tracks that drift away from their designed position and rails that become uneven due to some external factors, such as the frequent passage of heavy trains and deformation of the track bed [26], [27]. Track irregularities are position dependent and exist at all positions along a track even for a newly built highspeed line or railways after maintenance activities or renewal. Track irregularities substantially contribute to safety problems and further track deterioration. However, track irregularities are also distinguishable features that can facilitate localization



Fig. 2. Typical plots of the cross-level deviation measurements in three independent surveys over the same slab track section.

along a track, as demonstrated in the present research. That is, a track irregularity profile can be related back to an absolute position along a track on a digital map, i.e., the train position can be determined by comparing measured track irregularity samples to a digital map to find the best match, as illustrated in Fig. 1. Therefore, the present research addresses the following question through an experimental approach: *Are railway track irregularities useful features for matching-based train localization, and is it possible to achieve accurate positioning based on irregularity matching*?

The remainder of this paper is organized as follows: Section II introduces the track irregularity model and related measurement and discusses the positioning potential of track irregularity features. Thereafter, we introduce the experiment and analyze the results. Finally, we discuss the challenges of the train positioning system with regards to onboard track irregularity matching.

II. TRACK IRREGULARITY MODEL

Location-dependent track irregularities exist at all locations along a track and can be regarded as the track's fingerprint or texture. Train position can be estimated or refined by comparing the measured track irregularities with a predefined feature map, as illustrated in Fig. 1.

A. Track Irregularity Features

Several kinds of principal track geometric parameters are used to indicate track geometric quality or irregularity, including the alignment, longitudinal level, gauge, cross level or superelevation, and twist [26]–[28], the definitions of which can be found in [28]. Although staked out with demanding high accuracy in the construction and maintenance phases, real rails can never be perfect, with irregularities and distortions existing at all locations along a track even for a high-speed line. For example, Fig. 2 shows a typical plot of the crosslevel deviation measurements over the same track section of a newly built slab track of a high-speed line in China, measured by a track geometry measuring trolley (TGMT) [27] in three independent trials. It shows that irregularities with small magnitude indeed exist at the cross level even for a high-speed track and that a trolley gives repeatable responses to the track irregularities. In addition, these plots suggest that the cross-level irregularity correlates to longitudinal position, implying that the position of a rail vehicle can be estimated by correlating a previously mapped track with a vehicle's irregularity measurement sequences, transformed into spatial irregularity measurements.

It should be noted that cross-level plot above is used as an example to illustrate this issue, measurements of other kind of track irregularities have similar characteristics and also show good repeatability. According to the authors' experience in railway track geometry measurements, track irregularity has several inherent characteristics that make it useful for train localization:

- Track irregularities are position dependent and exist at all locations along a track, which can be regarded as a track's fingerprint or texture.
- Track irregularities or the vehicle's responses to track irregularities can be measured with sufficient accuracy using on-vehicle sensors, as discussed in the following subsection.
- Track irregularity textures are stable and change slowly with time, especially for the ballastless slab track, for example, which is to a large extent maintenance free [29, p.261-263], only small track irregularity change may occur over a year or even longer period of time.
- Track irregularities are distinguishable for parallel tracks, which would make it possible to identify which track the train is running on in the parallel track sections.

B. Track Irregularity Measurements

Localization by matching the track irregularity profile requires measuring the irregularity features in real time with sufficient accuracy. Track irregularity measurements obtained with onboard sensors or measuring systems are modeled as the superposition of real signals and disturbance terms as follows:

$$\tilde{I}_i = I_i(ID, s) + \delta I_i \tag{1}$$

$$\delta I_i = \delta I_{w,i} + \delta I_{v,i} + \delta I_{susp,i} + \delta I_{rest,i} \tag{2}$$

where \tilde{I}_i denotes the *i*-th irregularity measurement or the corresponding indirect responses and the subscript i denotes different types of track irregularity, such as the alignment, longitudinal level, cross level, twist, and gauge. $I_i(ID, s)$ refers to the actual irregularity, which is a function of the specific track ID and the longitudinal position s. It contains the repeatable, location-dependent components, which are useful signals for matching-based train location determination. δI_i represents a disturbance in the measurement from onboard sensors. $\delta I_{w,i}$ and $\delta I_{n,i}$ are disturbances due to the train weight and operating speed, respectively. The train imposes a load onto the track and makes the track irregularities slightly different from those in the unloaded state. For the slab track of a high-speed line, the difference between the loaded and unloaded conditions will be smaller. $\delta I_{susp,i}$ refers to the "wheelbase filtering" effect on the measurements, since the onboard sensors or measuring systems respond to the track irregularities through bogie wheelsets and the primary and secondary suspension systems. For example, the vehicle's pitch response acts as a low-pass filter to grade changes in the track. $\delta I_{rest,i}$ includes

other effects, such as the measuring noise and extremely short wavelength track defects and corrugations.

The precise measurement of track irregularities with onboard sensors requires carefully handling the disturbance terms incorporated in the measurement. Accurately separating the disturbance terms from the real track irregularities is complicated and has been studied in numerous previous works on track geometry condition inspection and monitoring, such as [20], [30], [31]. In this work, our objective is not to model or eliminate the disturbance terms individually, and we accept that the irregularity measurement is not perfect. Instead, we prove that the disturbance effect is not significant enough to filter out the measurement of the actual signal, i.e., $I_i(ID, s)$ is useful for determining a vehicle's position.

C. Correlation Between Track Irregularity and Attitude Response

According to the research on track geometry condition inspection technology, an additional filter or inverse model is necessary to identify the real signal components. In this research, we instead choose to measure some intermediate quantities or indirect responses that correlate to irregularities, because there is a strong correlation between track irregularities and the vehicle's attitude responses [24], [26]. In this case, measuring the attitude angles and gauge instead of measuring the irregularity directly would simplify the calculation.

For example, cross level refers to the difference in height of the two rails at a given mileage, which can be determined by measuring the angle between the running surface [28]. The cross level irregularity can be computed from the roll angle response measurement. The lateral and vertical irregularity can be computed by integrating the heading angle deviation and pitch angle deviation, respectively, with respect to the travel distance. It should be noted that the track gauge deviation can be directly measured using a distance sensor, and is not related to the vehicle's attitude response. Hereafter, the profile matching based on track irregularity measurement or the vehicle's responses to the track irregularity is named irregularity matching for short.

D. Track Irregularity Map

The train position in a railway network is defined by a unique track ID and a track length variable s. The origin of that length has to be defined for direction dir of the train related to the track. A positive direction points away from the origin, while a negative direction, towards the origin. The topological pose is a triplet that includes the track ID, length, and direction and defines the train position and attitude in topological coordinates unambiguously.

In practice, the map is organized in accordance with a list of tracks. Each track contains a unique track ID, connections and track data parameterized to a one-dimensional position s on the track. The geographic and geometric data are stored by supporting points of any kind of continuous graph representation. Many methods exist for representing a continuous function by discrete points. The simplest might be the polygonal line approximation, which interconnects points with lines. More



Fig. 3. Photograph of the experimental setup. (a) Overview of the maintenance train; (b) MEMS IMU mounted on the bogie platform; (c) IMUs mounted inside the cabin.

advanced methods for the geographic track representation use the spline approximation.

$$map = \{ID, s, dir, pos, Ir\}$$
(3)

$$query = \{ID, dist, dir, Ir\}$$
(4)

Therefore, this work examines whether the response of onboard sensors to track irregularities is repeatable and whether a disturbance in the measurement is significant relative to the real signals.

III. EXPERIMENT AND FEASIBILITY ANALYSIS

A train-borne experiment on a track maintenance car was conducted in November 2019 to study the possibility of determining the train position by matching the track irregularity feature. Datasets were collected using several GNSS/INS integrated systems of different accuracy, including a navigation-grade, a tactical grade, and two microelectromechanical system (MEMS) grade integrated system. The navigation-grade system, POS-A15, is manufactured by Leador Spatial Information Technology Corporation, Wuhan, China, and integrates a ring laser gyro-based inertial measurement unit (IMU) and a high-precision GNSS receiver. The typical tactical-grade system, POS-320, is manufactured by Beijing NAV Technology Co., Led, Beijing, China. This system integrates a tactical-grade IMU containing three closedloop fiber optic gyros and servo accelerometers and a NovAtel GNSS OEM 6 receiver. The MEMS GNSS/INS integrated system, named INS-Probe, was developed by the Navigation Group of the GNSS Research Center at Wuhan University. In this system, an MEMS IMU, ADI16465, from Analog Devices Inc. (ADI), MA, USA, is integrated and synchronized with a built-in GNSS receiver card from u-blox, Thalwil, Switzerland. Table I lists the specifications of the key equipment.

Fig. 3 shows a photograph of the experimental setup. Geodetic GNSS antennas were mounted on the roof of a train; one MEMS system, i.e., INS-Probe #2, was installed on the bogie platform; and the other systems were mounted inside the cabin, as shown in subplots (b) and (c). The lever arms,



Fig. 4. The train's trajectory in the test.

i.e., the vector from the IMU measurement center to the corresponding antenna phase center, were measured accurate to within 2 cm.

Fig. 4 shows the train's trajectory in the test. This path is approximately 12 km long and in an open-sky environment without significant signal obstacles, in which case the GNSS rover receivers operate in fairly favorable circumstances. A GNSS base station, the location of which is marked with a triangle, was set nearby and recorded raw observations simultaneously at 1 Hz to allow for carrier phase-based differential GNSS positioning processing. The baseline length between the rover receivers and the base station did not exceed 12 km. All systems recorded raw IMU measurements and GNSS observations at 200 Hz and 1 Hz, respectively. Different systems were synchronized to the GNSS time system. POS-A15 aided by GNSS real-time kinematic position in postprocessing is used to provide independent position reference, which is accurate to about 2 cm.

It should be noted that the reason why multiple different IMUs are used in the experiment are as follows: 1) We want to evaluate how accurate the IMUs should be for the proposed method, and whether a low-cost IMU can be used to implement the feature matching. 2) Two MEMS IMUs are mounted in different places to compare the different responses and evaluate whether the difference is significant when IMUs are mounted in different places. 3) Evaluate the similarity between the responses to the track irregularity from different IMUs.

To fully test the proposed method and draw a profound conclusion, 6 independent groups of datasets were collected according to the following procedure:

- (1) The train was kept stationary for approximately 300 s, allowing static alignment of the high-grade GNSS/INS system.
- (2) The train ran southward from the starting point to the end terminal, and then ran backward, *i.e.*, northward, without turning around, as depicted in Fig. 4, to collect datasets of run 1 and run 2.

POS-A15	Navigation-grade GNSS/INS integrated system
	Gyroscope in-run bias stability: 0.01 deg/h
	Angular random walk: 0.0022 deg/ \sqrt{h}
	Accelerometer in-run bias stability: 10 mGal
	Velocity random walk: 0.00075 m/s/ \sqrt{h}
POS-320	Tactical-grade GNSS/INS integrated system Gyroscope in-run bias stability: 0.25 deg/h
	Angular random walk: 0.1 deg/ \sqrt{h}
	Accelerometer in-run bias stability: 300
	mGal
	Velocity random walk: 0.03 m/s/ \sqrt{h}
INS-Probe	MEMS IMU, ADI16465
	Gyroscope in-run bias stability: 40 deg/h
	Angular random walk: 0.3 deg/ \sqrt{h}
	Accelerometer in-run bias stability: 1000
	mGal
	Velocity random walk: 0.1 m/s/ \sqrt{h}
Trimble NetR9	GNSS base station receiver
	Sampling rate: up to 50 Hz (configurable)
	RTK surveying performance:
	horizontal 8 mm + 1 ppm RMS
	vertical 15 mm + 1 ppm RMS
GNSS antenna	NovAtel GPS-702-GGL
	Signals tracked: GPS L1/L2, GLONASS L1/L2, L-band, BDS B1, Galileo E1
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Fig. 5. The train's speed profile for different runs.

- (3) Steps 1 and 2 were repeated under similar conditions to collect datasets of run 3 and run 4.
- (4) Steps 1 and 2 were repeated at different speeds with respect to the first four runs to collect datasets of run 5 and run 6 in order to evaluate the speed effects. Fig. 5 shows the speed profiles for each run.

A. Data Processing

1) Basic Data Processing: The basic data processing follows such a procedure: 1) Process the GNSS and IMU data from different systems to obtain an integrated position and attitude solution. 2) Resample the attitude sequences with respect to the travel distance. 3) Evaluate the matching accuracy based on the attitude response sequences.

The GNSS data from the base station and rover receivers were processed in the carrier-phase-based differential position-



Fig. 6. Typical attitude responses of the cabin-mounted POS-320 with respect to the travel distance.

ing mode in forward filtering, as the real-time kinematic (RTK) positioning works to provide a position with centimeter-level accuracy [32, p.635]. The IMU data from different IMUs were then fused with the GNSS position in a loosely coupled integration in the forward filtering processing to emulate the real-time data processing. The GNSS/INS processing provides position and attitude solutions, including the roll, pitch and heading responses to track irregularities.

The attitude solutions were then resampled and analyzed with respect to the travel distance. The travel distance for the attitude profile can be computed either by using the accurate GNSS/INS position or by integrating the GNSS/INS integrated velocity solutions with respect to time. The attitude measurements were decimated and interpolated at 0.1 m intervals to allow for the subsequent direct path-dependent comparison between the attitude measurements. Fig. 6 shows the entire attitude responses from POS-320, which used as an example, with respect to the travel distance.

The Pearson linear correlation coefficient, denoted by r_{xy} , is used as a measure of the similarity between the attitude responses and the attitude maps. The correlation coefficient r_{xy} of two discrete-time sequences containing *m* data points is calculated by:

$$r_{xy} = \frac{\sum_{i=1}^{m} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{m} (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^{m} (y_i - \bar{y})^2}}$$
(5)

where *m* is the sample size; x_i and y_i are the *i*-th data points of sequences $\{x\}$ and $\{y\}$, respectively; $\bar{x} = \frac{1}{n} \sum_{i=1}^{m} x_i$ is the sample mean; and \bar{y} refers to the sample mean of $\{y\}$. The correlation coefficient is a measure of the strength of the relationship or association of two sequences. The MATLAB function, *corr*, is used for its implementation.



Fig. 7. A typical section of the attitude measurements from the cabin-mounted POS-320 as a function of the path distance over 6 runs.

We now evaluate the matching accuracy by denoting the irregularity sequences and introducing certain notational conventions:

- $R_i(1, \dots, n)$ is the sequence of the *i*-th kind of background track irregularity (as described in Section II-A) with length *n*.
- $Q_i(1, \dots, n)$ is the query sequence of the *i*-th kind of measured irregularity with length n.

2) Multiple Datasets Processing: Note here that the background track irregularities have been previously mapped and that on-vehicle storage of the map is available. For the subsequent analysis, the attitude and position data from run 1 of the entire track are used as the background map, and the measurements from other runs are used as the in-vehicle measurement, hereafter called the query data or measured data. In addition, the train is assumed to be localized at a global scale with an initial position from other approaches, such as GNSS single-point positioning (SPP). Our work focuses on the precise local localization by locally matching the measured track irregularity profiles to the map. The data processing procedure is described below:

- (1) Extract the current query subsequence from the attitude measurements, Q_i , with a window of a given length, i.e., the *window length or query length*.
- (2) Section the map sequence to cover the same path distance as that covered by the query subsequence.
- (3) Calculate the Pearson linear correlation coefficient of the query subsequence and the map subsequence.
- (4) Shift the map subsequence by one sampling distance interval, i.e., 0.1 m.
- (5) Repeat steps 2-4 until the end of the search area has been reached. The search range for matching is set to 40 m, i.e., 20 m in the forward and backward

longitudinal directions, respectively. 20 m is the assumed initial global positioning error from other sensors.

- (6) Select the maximum correlation coefficient for the current query subsequence, and determine the longitudinal position from the map related to this maximum correlation coefficient.
- (7) Calculate the matching-based localization error of the current query subsequence by comparing the position determined from step 6 with the reference ground truth, which is accurate to within 2 cm.
- (8) Move the selection window, and repeat steps 1-7 until the end of Q_i has been reached.
- (9) Repeat steps 1-8 to evaluate the localization accuracy for different signal sources, including the roll, pitch, and heading responses.
- (10) Repeat steps 1-9 to evaluate different-grade GNSS/INS integrated systems, including a navigation-grade, a tactical-grade and two low-cost MEMS systems, as mentioned in the experimental description.

The search range in step 5 determines how far away from the currently assumed position the algorithm will search for similar subsequences in the map. If the currently assumed position is not very certain, then a larger search range is needed. However, increasing the search range also increases the risk of false matches. The latter is a potential issue, especially if repeating strong features occur in the track environment. To reduce the search range, an INS and odometer are integrated as the dead reckoning (DR) system to provide a coarse initial position for local matching in this structure.

B. Result Analysis

For the first step, we examine the repeatability of attitude response in 6 different runs over the same track section. Take the results from POS-320 as an example, the plots are shown in Fig. 7. This figure shows that the attitude measurements correlate to the travel distance, *i.e.*, the longitudinal position along the track, and the responses in different runs show good repeatability. It can be seen from this figure that the difference between the heading angle sequences of multiple runs is larger than that for the roll and pitch angle responses. The reason is that the roll and pitch angles of the GNSS/INS integrated system are known more accurate than heading, because errors in the heading angle and the vertical component of gyro bias are weakly estimable and observable in this condition [33]. Thus, the heading matching accuracy is expected to be not as good as using the roll and pitch angles, which can be validated in the subsequent analysis. The roll angle plots tend to gradually arise at the end of the curves, because the train traveled at a transition section where the real roll angle change linearly with respect to the travel distance.

Comparing the plots from the forward direction (runs 1, 3 and 5 in the upper three panels) and reverse direction (runs 2, 4 and 6 in the lower three panels), it can be seen that the attitude responses show good similarity when the train traveled in the same direction, while slight difference can be observed between the plots from forward direction and reverse directions. The attitude responses from run 5 and run 6 are slightly different from other runs either in the forward or reverse directions, the possible reason is that run 5 and 6 have different travel speed with respect to the other four runs, as depicted in Fig. 5. On the other hand, the speed difference between run 5 and 6 with respect to the other four runs is relatively small in our experiment, it is not easy to draw a rigorous conclusion on how speed variation influences the attitude responses. Fortunately, in a general case, for a given railway line, there is a corresponding operation speed graph and trains are required to run at similar speed at the same sections. Thus, the impact of the speed variation on the matching accuracy would be small in the real applications.

It can be seen from Fig. 7 that the pitch angle responses from multiple runs have the best similarity among the three different type of attitude angles. In the following, we take the pitch angle responses of the POS-320 as an example to evaluate the matching accuracy. Fig. 8 shows the matching error plots obtained using the pitch responses of POS-320 for different runs, in which case the query length is 100 m, with 1000 data points for matching. These initial estimation error plots show promising results, as most of the errors do not exceed 1 m in run 3 and run 5, while the error is apparently much greater in the other runs, *i.e.*, runs 2, 4 and 6. Because we chose the pitch angle response from run 1 as the background map for matching, runs 3 and 5 have the same travel direction as that of the data collected for the map, while runs 2, 4 and 6 have the opposite travel direction, as mentioned before. Smaller errors are observed in run 3 than those in run 5 because run 3 has a speed profile similar to that of the map, while run 5 has a different speed profile. Note that similar analysis can be made using roll or heading angle responses and for other IMUs.

The performance of subsequence matching is known to be affected by the length of the query sequence. If the query



Fig. 8. Matching errors (absolute value) using the pitch profile with a query length of 100 m from POS-320. The pitch angle measurements from run 1 were used as the map.

sequence is too short, it will not contain sufficient information for good matching; if it is too long, the results will suffer from feature distortions and errors in the travel distance measurement, which is, for example, computed by integrating speed with respect to time. Here, a common question for the present research is how long a query subsequence should be to achieve a reliable and robust matching. We attempt to address this issue by conducting a statistical analysis of the positioning errors following such a procedure: 1) Calculate the root mean square (RMS) value of the matching error sequences by using the pitch responses for a given length. 2) Repeat step 1 for different query lengths. 3) Repeat steps 1-2 by using the roll and heading angle responses, respectively. 4) Repeat steps 1-3 to evaluate the performances of different IMUs.

Fig. 9 shows the matching accuracy convergence plots with respect to the query length. The plots show that the matching accuracy achieved by using the pitch angle responses converges for all cases as long as the query length reaches 50 m. The accuracy plots for roll angle also converge for all cases except for the bogie-mounted INS-Probe. The accuracy obtained using the heading angle responses only converge for the high-grade IMUs, e.g., POS-A15 and POS-320, while they do not seem to converge for both the cabin- and bogiemounted MEMS IMUs. The possible reason is that the heading angle from the low-cost MEMS system drifts more significantly and is not accurate enough for precise matching. The heading matching errors are larger than those obtained using the roll and pitch angle responses for POS-320 and POS-A15 because the roll and pitch accuracies of GNSS/INS integrated systems are better than the heading accuracy, as discussed previously in interpreting Fig 7. Comparing the results from the bogie-mounted and cabin-mounted INS-Probe, we find that the train's bogie platform and cabin have different dynamic attitude responses to track irregularities due to the different suspension systems.

We can cautiously conclude from this figure that the localization accuracy converges to approximately 0.4 m (RMS) when using the roll and pitch alone for matching for POS-

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Fig. 9. Matching errors (root mean square) as a function of the query subsequence length using the roll, pitch and heading responses.

A15, POS-320 and the cabin-mounted INS-Probe. Only highprecision systems, i.e., POS-320 and POS-A15, show potential for train positioning when the heading response is used for matching. Pitch angle response converge for all cases for different platform, different speed and different IMUs. Therefore, pitch and roll responses to the track irregularities are the two most promising signals for the accurate train positioning by matching, and can be performed using low-cost IMUs.

In practice, the IMUs used to create the background map may be different from the onboard IMUs which provide the real-time attitude responses as the query sequences. For example, we may use a high-grade GNSS/INS system to collect the map dataset and use the low-cost IMU on board to perform real-time positioning by matching. Therefore, we now study the influence in this condition by evaluating the performance of matching the query responses from POS-320 and the lowcost cabin-mounted INS-Probe to the background map data from high-precision integrated system POS-A15.

Fig. 10 shows the matching error sequences obtained by using the pitch responses of POS-320 and INS-Probe from run 1, run 3 and run 5, which have the same travel direction. The query length is set 100 m. The localization estimation errors show that most of the errors do not exceed 1 m in the 3 runs. A smaller error can be observed in runs 1 and 3 than that in run 5, which occurs because runs 1 and 3 have speed profiles similar to that of the map, while run 5 has a different speed profile. Compare the upper panels of Fig. 10 and Fig. 8, it can be seen that the accuracy plots by matching the same pitch responses to different maps created by using different grade IMUs are not significantly different. This result is promising since it means that we have more flexibility in



Fig. 10. Matching errors (absolute value) using pitch angle with a 100 m query length. The background map comes from the solutions of POS-A15 in run 1.

the choice of the IMU when creating and updating the maps. Comparing the two subplots in Fig. 10, we also find that the low-cost INS-Probe and POS-320 have similar matching accuracies, which means that *the low-cost MEMS system also has the potential for precise matching-based positioning*.

IV. PRACTICAL IRREGULARITY MATCHING-BASED LOCALIZATION

From the conception and verification to the actual positioning system implementation of the proposed method, there is



Fig. 11. Concept of the train localization prototype based on track geometry feature matching.

still a issue that must be addressed. In the following section, we first describe a possible design of the positioning system using the track irregularity matching, and then discuss in detail several issues involved in implementing this system.

A. Concept of the System Design

Track irregularity matching is not a standalone navigation technique, we propose to use it as part of an integrated navigation system. Fig. 11 depicts the overview of a possible multisensory positioning prototype aided by track irregularity matching. The proposed system mainly contains three parts: a track-irregularity-sensing part, a profile-matching part and a state estimator for the multisensory system. The track irregularity matching works as a part of the multisensory train positioning system to enhance its robustness, integrity and performance.

The track-irregularity-sensing part is designed to measure a train's responses to the principal track irregularities, such as attitudes, gauge deviation or other kinds of track irregularities. For example, the track gauge can be accurately measured independently with a pair of laser scanners even under high-speed operation conditions and then used for precise profile matching. However, maintaining a full track irregularity recording system on a vehicle for train positioning purposes is an expensive option. The most promising and cost-efficient approach is to measure the train's attitude response to track geometric irregularities by using a low-cost cabin-mounted IMU, as analyzed in previous sections.

The onboard IMU, on the other hand, is used to provide velocity, coarse position solutions through a mechanization procedure to determine the relative positions of the measured attitude responses. Then, train's response signal to track irregularity is synchronized with the travel distance and resampled with a constant distance interval to produce the query subsequences. The track irregularity sensing system should sample at a sufficiently high data rate in the time domain, specifically for a high-speed train. For example, if a train moves at a speed of 80 m/s and the gauge sensor samples, at a frequency of 400 Hz, then the distance interval between two samples would be 0.2 m.

For the matching part, the track irregularity or response sequence is matched up to a predefined background map of track irregularities to determine the train position. In the



Fig. 12. Localization errors of the INS aided by pitch profile matching from POS-320.

real-time implementation, we need to store the measurement of track irregularity response in the past short period of time to generate the query sequence. The problem of track irregularity matching can be broken down into two phases: global localization and local tracking. Global localization attempts to estimate the position of the vehicle in the initial phase, during which the vehicle can be present anywhere on the map. The feature matching systems are initialized with an approximate position solution to determine which region of the database to search. Limiting the database search area minimizes the computational load and the number of instances where more than one possible match exists between the measured features and those in the database. The global initial position in this research is provided by either an IMU/odometer-integrated system or a GNSS position solution, and the matching algorithm focuses on precise local tracking.

In the state estimation part, the matched position is sent to the state estimator for multisensor data fusion as the external observation to improve the positioning performance and obtain the final optimal position estimates. The estimated inertial sensor bias are then fed back to the INS mechanization procedure.

B. Preliminary Result of the INS Aided by Irregularity Matching

In this section, we preliminarily evaluate the positioning accuracy of the INS aided by matching to enhance the previous conclusions on the irregularity matching. The matched positions and the raw IMU data, e.g. from POS320, are fused to perform the integrated navigation through a Kalman filter (KF), and the KF outputs are then compared with the reference solution from the high-precision system POSA15 in postprocessing to evaluate the accuracy. Details on the integration algorithm and Kalman filter design can be found in [34]–[36].

Fig. 12 depicts the localization estimation error plots in runs 3 and 5. It shows that the positioning solution is accurate to 1 m at a 99.7% confidence level. The positioning error in run 3 seems smaller than that in run 5, because run 3 has a speed profile similar to that of the map, i.e. run 1, as discussed previously. Comparing the upper panel of Fig. 8 with Fig. 12, we find that the positioning accuracy of the INS aided by matching is consistent with that of matching. It means that



Fig. 13. Localization error of the DGNSS/INS integrated solution from POS320.



Fig. 14. Cumulative distribution function plots of the localization errors of different methods using POS-320.

the track irregularity matching contributes to maintaining the absolute positioning accuracy and preventing the INS accuracy drift, while the INS enables the integrated system to output accurate positioning solutions at a high data rate. Returning to Fig. 12 and upon closer observation, we notice that there are small steps in the error curves. The most likely reason is the matching error since similar steps can also be found in Fig. 8 and 10. It should be noted that in the present research we concentrate on studying the potential of trains' response to railway track irregularities in train localization. Details on the data fusion algorithm for INS aided by track irregularity matching and a comprehensive evaluation of its performance will be presented in a follow-up paper.

For comparison, we process the same GNSS and IMU data from POS320 using the multi-sensor data fusion algorithms that have been practically adopted in the railway system, including the code-based differential GNSS positioning (DGNSS) aided INS and the SPP aided INS. As an example, Fig. 13 shows the DGNSS/INS localization errors of multiple tests. Fig. 14 depicts the cumulative distribution function (CDF) plots of the localization errors of SPP/INS, DGNSS/INS integration, irregularity matching only and INS aided by matching. Table II lists the statistic values of location errors for different positioning algorithms. The reported typical

 TABLE II

 STATISTICS OF LOCATION ERRORS FOR DIFFERENT METHODS

Method	RMS (m)	95% (m)	99.7% (m)
SPP/INS	1.89	3.31	3.78
DGNSS/INS	0.44	0.85	1.42
matching only	0.39	0.69	0.96
INS/matching	0.37	0.78	1.00

positioning accuracy of existing multisensory system is as follows: the maximum RMS error is approximately 1.5 m in post-processing, and the mean RMS error has a wider range from 0.135 to 1.04 m [1], [2]. Therefore, comparisons show that the track irregularity matching is promising to achieve equivalent accuracy to the code-based differential GNSS/INS integrated solution, and has great potential in precise train positioning application. On the other hand, compared with the GNSS-enabled approaches, the irregularity matching-enabled method is less likely interfered by signal blockages.

C. Discussion

In this research, we have validated the feasibility of track irregularity matching for enhancing the train positioning performance by using the case study method. The results show that the train's roll and pitch angle responses to track irregularities are repeatable and would significantly contribute to precise train positioning. These response signals can be measured even using a low-cost cabin-mounted MEMS IMU. A critical issue on this conclusion may be that how credible is it when obtained through an experimental approach. As mentioned previously, we find that there are some limitations in our experiment. For example, we cannot comprehensively verify the influence of the changes of travel speed and train load, because it is difficult to do such experiments on the operating line. But we have found that many previous research work can provide strong support for the conclusions we have reached, especially by the research work on railway track condition inspections.

This conclusion from the case study is supported by the European railway standard. According to the European standard on the track condition inspection vehicles [28], the degree of agreement between successive measurements of the same track parameters from track recording vehicles under varying conditions, including variations in speed, different vehicle orientations, and different environmental conditions, should comply with the specified accuracy requirements. Therefore, it implies that track irregularities or the corresponding responses can technically be measured with a sufficient accuracy for matching-based train localization. Weston et. al proved in his research [20], [30], [31] that lateral and vertical track irregularity can be accurately measured by inertial sensors, i.e., gyroscope and accelerometer, onboard the inservice railway vehicles; and the measurements shows good repeatability. These and similar work have provided a strong support for our conclusion in this paper.

In this research, the background track irregularity map was assumed available in the data analysis. However, the creation and updating of the background database are crucial issues for any map-matching-based positioning application. The proposed track irregularity matching-enabled positioning method also relies highly on the track irregularity map. We provide several possible strategies that may release the problem in data source for map construction and update: 1) The track geometric and irregularity measurement information in the railway construction phase can be used as the initial map. This method is quite practical for a high-speed line. 2) Dedicated track geometry condition inspection vehicles can be used to create and update the existing track irregularity map. 3) In the future, any in-service train equipped with a track irregularity sensing system or sensors can be used to collect related information and update the map via a crowdsourcing approach. However, things will be more complicated to integrate the track irregularity matching into a practical train positioning system [22], considering the map data structure definition, storage, map version management, map download from track

For a real train localization case, the track irregularity measurements from an onboard measuring system would suffer from amplitude and distance scaling problems and signal distortions. A more robust subsequence matching algorithm that can handle this problem should be chosen for realtime localization applications in terms of both the matching reliability and time efficiency. In addition, the travel distance measurement errors influence the subsequence matching, the quantitative evaluation of its effects, and the real-time routine; the implementation issues are addressed in our research work in the near future.

trackside to the train on-board, map check, and risk of

integrity et. al.

In practice, similar to any other feature matching approaches, the proposed method also may fail to provide the matched navigation information either if there are insufficient features in the query sequence or the database, or due to ambiguous features. Thus, track irregularity matching is not a standalone navigation technique, it is only used as part of an integrated navigation system. To prevent the matching failure or wrong matching from corrupting the integrated navigation solution in a filtered architecture, measurementinnovation based fault detection and integrity-monitoring techniques should always be used. More details can be found in the textbook [17, p.706].

V. CONCLUSION

This work demonstrates the estimation of a train's position along a track by matching the track irregularity measurements or the responses to the track irregularity by using onboard sensors to the corresponding predefined map. Train-borne experimental results show that the attitude responses of the onboard IMUs, *i.e.*, a kind of indirect measurement of the track irregularity, are location dependent and show good repeatability. The results show that the positioning solution can be accurate to within 0.4 m (RMS) even using the roll and pitch angle responses from cabin-mounted low-cost MEMS IMUs. The matching-based positioning accuracy converges when the query length reaches 50 m. Therefore, the direct onboard measurement of track irregularity or the onboard measurement of the responses of the train to track irregularity has great potential in precise train localization applications. The proposed method is expected to be exploited as a subsystem of the current multisensory track positioning system to enhance the localization performance to benefit autonomous train driving in the near future.

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REFERENCES

- J. Otegui, A. Bahillo, I. Lopetegi, and L. E. Diez, "A survey of train positioning solutions," *IEEE Sensors J.*, vol. 17, no. 20, pp. 6788–6797, Aug. 2017.
- [2] K. Kim, S.-H. Kong, and S.-Y. Jeon, "Slip and slide detection and adaptive information sharing algorithms for high-speed train navigation systems," *IEEE Trans. Intell. Transp. Syst.*, vol. 16, no. 6, pp. 3193–3203, Dec. 2015.
- [3] M. Malvezzi, G. Vettori, B. Allotta, L. Pugi, A. Ridolfi, and A. Rindi, "A localization algorithm for railway vehicles based on sensor fusion between tachometers and inertial measurement units," *Proc. School Mech. Eng. F, J. Rail Rapid Transit*, vol. 228, no. 4, pp. 431–448, 2014.
- [4] P. Ernest, R. Mazl, and L. Pfeucil, "Train locator using inertial sensors and odometer," in *Proc. IEEE Intell. Vehicles Symp.*, Jun. 2004, pp. 860–865.
- [5] M. Marinov, S. Hensel, and T. Strauß, "Eddy current sensor based velocity and distance estimation in rail vehicles," *IET Sci., Meas. Technol.*, vol. 9, no. 7, pp. 875–881, Oct. 2015.
- [6] C. Reimer, F. J. Müller, and E. L. V. Hinuber, "INS/GNSS/odometer data fusion in railway applications," in *Proc. DGON Intertial Sensors Syst. (ISS)*, Sep. 2016, pp. 1–14.
- [7] B. Allotta, L. Pugi, A. Ridolfi, M. Malvezzi, G. Vettori, and A. Rindi, "Evaluation of odometry algorithm performances using a railway vehicle dynamic model," *Vehicle Syst. Dyn.*, vol. 50, no. 5, pp. 699–724, May 2012.
- [8] S. Damy, A. Majumdar, and W. Y. Ochieng, "GNSS-based high accuracy positioning for railway applications," in *Proc. Int. Tech. Meeting The Inst. Navigat.*, Feb. 2016, pp. 1003–1014.
 [9] D. Liu, W. Jiang, B. Cai, J. Wang, and W. Shangguan, "A tightly-coupled
- [9] D. Liu, W. Jiang, B. Cai, J. Wang, and W. Shangguan, "A tightly-coupled GNSS/INS/MM integrated system based on binary search algorithm for train localization applications," in *Proc. 32nd Int. Tech. Meeting Satell. Division Inst. Navigat. (ION GNSS+)*, 2019, pp. 1272–1283.
- [10] B. Cai and X. Wang, "Train positioning via integration and fusion of GPS and inertial sensors," WIT Trans. Built Environ., vol. 50, pp. 1217– 1226, Aug. 2000.
- [11] J. Otegui, A. Bahillo, I. Lopetegi, and L. E. Diez, "Evaluation of experimental GNSS and 10-DOF MEMS IMU measurements for train positioning," *IEEE Trans. Instrum. Meas.*, vol. 68, no. 1, pp. 269–279, Jan. 2018.
- [12] J. Marais, J. Beugin, and M. Berbineau, "A survey of GNSS-based research and developments for the European railway signaling," *IEEE Trans. Intell. Transp. Syst.*, vol. 18, no. 10, pp. 2602–2618, Oct. 2017.
- [13] M. Lauer and D. Stein, "A train localization algorithm for train protection systems of the future," *IEEE Trans. Intell. Transp. Syst.*, vol. 16, no. 2, pp. 970–979, Apr. 2015.
- [14] O. Heirich, B. Siebler, and E. Hedberg, "Study of train-side passive magnetic measurements with applications to train localization," *J. Sensors*, vol. 2017, pp. 1–10, 2017.
- [15] O. Heirich, A. Lehner, P. Robertson, and T. Strang, "Measurement and analysis of train motion and railway track characteristics with inertial sensors," in *Proc. 14th Int. IEEE Conf. Intell. Transp. Syst. (ITSC)*, Oct. 2011, pp. 1995–2000.

- [16] K. Gerlach and M. M. zu Horste, "A precise digital map for GALILEObased train positioning systems," in *Proc. 9th Int. Conf. Intell. Transp. Syst. Telecommun.*, (*ITST*), Oct. 2009, pp. 343–347.
- [17] P. D. Groves, Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems, 2nd ed. Norwood, MA, USA: Artech House, 2013.
- [18] S. S. Saab, "A map matching approach for train positioning. I. Development and analysis," *IEEE Trans. Veh. Technol.*, vol. 49, no. 2, pp. 467–475, Mar. 2000.
- [19] S. S. Saab, "A map matching approach for train positioning. II. Application and experimentation," *IEEE Trans. Veh. Technol.*, vol. 49, no. 2, pp. 476–484, Mar. 2000.
- [20] P. Weston, C. Roberts, G. Yeo, and E. Stewart, "Perspectives on railway track geometry condition monitoring from in-service railway vehicles," *Int. J. Vehicle Syst. Dyn.*, vol. 53, no. 7, pp. 1063–1091, 2015.
- [21] O. Heirich, P. Robertson, A. C. Garcia, and T. Strang, "Bayesian train localization method extended by 3D geometric railway track observations from inertial sensors," in *Proc. 15th Int. Conf. Inf. Fusion* (*FUSION*), Jul. 2012, pp. 416–423.
- [22] O. Heirich, P. Robertson, and T. Strang, "RailSLAM–localization of rail vehicles and mapping of geometric railway tracks," in *Proc. IEEE Int. Conf. Robot. Autom.*, May 2013, pp. 5212–5219.
- [23] O. Heirich, A. Steingass, A. Lehner, and T. Strang, "Velocity and location information from onboard vibration measurements of rail vehicles," in *Proc. 15th Int. Conf. Inf. Fusion (FUSION)*, Jul. 2013, pp. 1835–1840.
- [24] A. Dean, R. Martini, and S. Brennan, "Terrain-based road vehicle localisation using particle filters," *Veh. Syst. Dyn.*, vol. 49, no. 8, pp. 1209–1223, 2011.
- [25] J. A. Dean, "Terrain-based road vehicle localization using attitude measurements," Ph.D. dissertation, College Eng., Pennsylvania State Univ., State College, PA, USA, 2008.
- [26] Q. Chen, X. Niu, Q. Zhang, and Y. Cheng, "Railway track irregularity measuring by GNSS/INS integration," *Navigation*, vol. 62, no. 1, pp. 83–93, Mar. 2015.
- [27] Q. J. Chen *et al.*, "A railway track geometry measuring trolley system based on aided INS," *Sensors*, vol. 18, no. 2, p. 26, 2018.
- [28] Railway Applications. Track. Track Geometry Quality. Characterization of Track Geometry, Standard BS EN 13848-1:2003, British Standard, 2008.
- [29] C. Esveld, *Modern Railway Track*. Zaltbommel, The Netherlands: MRT-Productions, 2016.
- [30] P. F. Westeon, C. S. Ling, C. Roberts, C. J. Goodman, P. Li, and R. M. Goodall, "Monitoring vertical track irregularity from in-service railway vehicles," *Proc. Inst. Mech. Eng.*, *F, J. Rail Rapid Transit Inst. Mech. Eng.*, vol. 221, no. 1, pp. 75–88, Jan. 2007.
- [31] P. F. Weston, C. S. Ling, C. J. Goodman, C. Roberts, P. Li, and R. M. Goodall, "Monitoring lateral track irregularity from in-service railway vehicles," *Proc. Inst. Mech. Eng., F, J. Rail Rapid Transit*, vol. 221, no. 1, pp. 89–100, Jan. 2007.
- [32] P. Teunissen and O. Montenbruck, Springer Handbook of Global Navigation Satellite Systems. Cham, Switzerland: Springer, 2017.
- [33] S. Hong, H.-H. Chun, S.-H. Kwon, and M. H. Lee, "Observability measures and their application to GPS/INS," *IEEE Trans. Veh. Technol.*, vol. 57, no. 1, pp. 97–106, Jan. 2008.
- [34] N. El-Sheimy and A. Youssef, "Inertial sensors technologies for navigation applications: State of the art and future trends," *Satell. Navigat.*, vol. 1, no. 1, pp. 1–21, Dec. 2020.
- [35] Q. Chen, Q. Zhang, X. Niu, and J. Liu, "Semi-analytical assessment of the relative accuracy of the GNSS/INS in railway track irregularity measurements," *Satell. Navigat.*, vol. 2, no. 1, pp. 1–16, Dec. 2021.
- [36] X. Niu *et al.*, "Development and evaluation of GNSS/INS data processing software for position and orientation systems," *Surv. Rev.*, vol. 47, no. 341, pp. 87–98, 2015.



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